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FINITE - ELEMENT ANALYSIS OF EDGE EFFECTS IN ANGLE - PLY COMPOSITE LAMINATES

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A three-dimensional finite-element modelling and analysis technique has been developed and used in the evaluation of edge stresses in angle-ply composite laminates. Excellent results have been obtained for the initial application to a four-ply ±45° laminate in tension, and the method is potentially applicable to holes, notches and other boundaries of arbitrary geometry.

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INTRODUCTION

The optimum design of aircraft structures and the maximum exploitation of the advantageous properties of advanced fiber reinforced materials require detailed knowledge of the stress fields within structural elements. Of particular importance are stress concentrations, for it has been indicated that they are of a different nature in composites than in conventional materials. Under reference (a), the NAVAIRDEVCEN (Naval Air Development Center) has conducted a program to evaluate the stress concentrations in composite structures.

The most basic form of stress concentration in a composite laminate is the occurrence of interlaminar shear and normal stresses near free edges, accompanied by variations in in-plane stresses. This stress concentration, often referred to as the edge effect, is caused by the heterogeneity of the laminate, and is, therefore, a characteristic more of the material than of the geometrical configuration of a structural element. The resulting perturbation of the stress field decays within a distance from the edge on the order of the laminate thickness, but may be sufficient to cause delamination and subsequent failure.

Since conventional laminate theory does not account for the edge effect, an alternate method is needed to analyze this localized phenomenon. Several approaches to the special case of a straight edge, using a variety of analytical and numerical methods, have been published. (References (b) through (f)). However, the methods used are either two-dimensional or are applicable only to straight edges. The effort at the NAVAIRDEVCEN has been the development, using NASTRAN, of a three-dimensional finite element modelling and analysis technique which can be applied to edges of arbitrary geometry, including holes and notches, and to bonded joints. The technique developed, and its application to the problem of a four-ply ±45° laminate in tension are described in this report. The results of the analysis of this elementary case are essentially identical to those obtained by other investigators and indicate that the interlaminar shear stress between the +45° and -45° plies may be sufficient to cause delamination.

PRELIMINARY ANALYSIS

Because of the availability of solutions by other methods, the finite element procedure has been first applied to the free edge of a four-ply, ±45° boron-epoxy laminate in tension, as shown in Figure 1. It is postulated that in the middle region of the laminate, away from the areas of load introduction, there is no variation of stress, strain or lateral and transverse displacement in the longitudinal (x) direction (parallel to the edge); only the variation of longitudinal displacement ($\partial u/\partial x = \epsilon_x$) is non-zero. In addition, by introducing compatibility relations from reference (g) and setting all partial derivatives with respect to x equal to zero, it is concluded that ϵ_x is a linear function of y and z of the form Ay + Bz + C. However, since the laminate and loading are symmetric, the linear terms must be zero, and therefore, ϵ_x is constant over the cross section. Also because of symmetry, there are no shear stresses on the mid-plane of the laminate, nor is there any displacement normal to it. The most important consequence of the lack of variation in the longitudinal direction and of the problem symmetry is that only two plies of a thin slice normal to the edge need to be modelled and analyzed. In addition, since the edge effect decays by 97 percent within a distance equivalent to the laminate thickness (four plies), the region to be analyzed extends only six ply thicknesses from the edge, to where the stress state can be determined by conventional laminate theory. The region to be analyzed, and the applicable stress, strain and displacement conditions are shown in Figure 2.

FINITE ELEMENT MODELLING

The grid point array of the finite element model is shown in Figure 3. To facilitate modelling the orthotropic material, the mesh is square in planform, subdivided to one-eighth ply thickness near the edge, where stress gradients are greatest. A single ply of the laminate is represented by four planes of grid points, as indicated by the shaded region in Figure 3. The grid points are interconnected by one- and two-dimensional elements -- rods, shear panels and quadrilateral membranes -- which together simulate the three-dimensional elastic behavior of the orthotropic material.

An enlarged view of the shaded region of Figure 3 is shown in Figure 4. The boron-epoxy material is modelled as two layers of epoxy separated by a fibrous layer whose thickness is equal to the fiber diemeter. The properties of the fibrous layer (Table I) are derived from those of the unidirectional material and the epoxy by means of a three-dimensional laminate analysis. Since each of the layers is orthrotropic, its elastic behavior can be described by nine independent stiffness coefficients (Figure 5). The stiffness represented by each coefficient is provided in the finite element model by one or more of the elements. For example, the interlaminar membranes, which are aligned parallel and normal to the fiber directions, contribute to the stiffness represented by the coefficients C_{13} and C_{23}. The transverse normal stiffness, C_{33}, and the interlaminar shear stiffnesses, C_{44} and C_{55}, are provided by the rods and shear panels.

LOADING AND CONSTRAINTS

The known stress and displacement conditions applicable to the analysis have been shown in Figure 2 and the constraints imposed on the model to simulate the strain/displacement conditions are listed in Table II. Zero displacement across the laminate mid-plane, the lack of variation of displacement in the longitudinal direction, and the uniform normal strain, ϵ_x, are enforced by means of single- and multi-point constraints. At the interior end of the model ($23h/4 \leq y \leq 6h$), additional constraints ($\delta U/\delta Y = \delta U/\delta Z = \delta V/\delta Z = \delta W/\delta Y = 0$) are applied to simulate the laminate theory solution. No forces are input. Instead, the unknown grid point forces are generated by NASTRAN as constraint forces in response to a sufficient number of known displacement conditions.

STRESS RECOVERY

To calculate the stresses in the individual fibrous and epoxy layers, the element stresses are transformed as necessary into a common coordinate system (laminate coordinates) and added together. For example, if the longitudinal normal stress, σ_x is expressed as

$$\sigma_x = C_{11}\epsilon_x + C_{12}\epsilon_y + C_{16}\gamma_{xy} + C_{13}\epsilon_z$$

the first three terms are equivalent to the average of the stresses, σ_x , from the upper and lower in-plane membranes and the third term is the sum of the stresses (transformed) of the interlaminar membranes. The stresses obtained by this method are applicable for the middle of each layer. In-plane ply stresses are obtained as the sum of the fibrous and epoxy layer stresses, each weighted in proportion to its thickness.

$$\sigma_i^{ply} = \frac{1}{t^{ply}}\left(\sigma_i^{epoxy} \times t^{epoxy} + \sigma_i^{fib.} \times t^{fib.}\right)$$

RESULTS AND DISCUSSION

The in-plane ply stresses, σ_x and τ_{xy} , and the interlaminar shear stress, τ_{xz} , in the adjoining epoxy layers of the two plies are plotted versus distance from the laminate edge in Figure 6 and are essentially identical to those obtained by other investigators. The remaining stresses, σ_y , σ_z and τ_{yz} , which arise primarily from material dissimilarities between the fibrous and epoxy layers, are two orders of magnitude smaller. At the laminate edge, where the in-plane shear stress, τ_{xy} , is zero, the longitudinal tensile stress, σ_x , is the same as would occur in unidirectional material loaded to the same normal strain, ϵ_x , at 45 degrees to its fiber direction, and is 20 percent lower than in the interior. In order to maintain equilibrium, the rise of the in-plane shear, τ_{xy}, and its asymptotic approach to the value predicted by laminate theory are accompanied by a decay of the interlaminar shear, τ_{xz} , from its maximum value at the edge. For each of these stresses, the influence of the free edge decays by 97 percent within four ply thicknesses. From an attempt to fit the data obtained from the finite-element analysis to analytical forms, it has been found that the variation of the major stresses in the direction normal to the edge can be very precisely described by simple exponential functions, as shown below:

$$\sigma_x = \bar{\sigma}_x - (\bar{\sigma}_x - \sigma_x^{o})\, e^{-\frac{7y}{8h}}$$

$$\tau_{xy} = \bar{\tau}_{xy}\left(1 - e^{-\frac{7y}{8h}}\right)$$

$$\tau_{xz} = \tau_{xz}^{o}\, e^{-\frac{7y}{8h}}$$

where h is the ply thickness, and the superscripted bar and zero denote values in the interior and at the edge, respectively.

The magnitude of the interlaminar shear stress at the edge, about equal to the inplane shear predicted by laminate theory for the interior, is most important, as the occurrence of so high a stress in the matrix may cause delamination and redistribution of stress leading to failure. This has been experimentally identified as a mode of failure (reference (h)) and some investigators (reference (i)) have applied fracture mechanics methods to quantitatively define failure loads for various angle-ply laminates.

CONCLUSIONS AND RECOMMENDATIONS

1. Finite element methods can be used in ply-by-ply analyses of local edge effects in composite laminates and will yield valid results. In the analysis described above, only the simplest one- and two-dimensional constant-strain elements were used. Even better results should be obtainable with simpler models by employing newly-developed parametric solid elements.

2. By applying supplementary analytical or numerical methods, it is possible to limit the region which must be modelled to only a small part of a structural element, as demonstrated above. Therefore, a finite-element technique similar to that used in the analysis of the straight edge is potentially applicable to boundaries of arbitrary geometry.

3. The edge effect stresses resulting from the analysis presented may be sufficient to cause delamination and subsequent redistribution of stresses within the plies.

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TABLE I

MATERIAL PROPERTIES

	COMPOSITE	FIBROUS LAYER	EPOXY LAYER
E_{11}, msi	20.00	26.30	0.607
E_{22}, msi	2.10	2.56	0.607
E_{33}, msi	2.10	3.89	0.607
G_{12}, msi	0.85	1.05	0.226
G_{13}, msi	0.85	1.05	0.226
G_{23}, msi	0.85	1.05	0.226
ν_{12}	0.21	0.20	0.340
ν_{13}	0.21	0.15	0.340
ν_{23}	0.21	0.11	0.340

TABLE II

CONSTRAINTS APPLIED TO EDGE EFFECT MODEL

CONDITION*	REGION WHERE APPLIED+
$u = 0$	$x = 0$; $y = \frac{23h}{4}$, $6h$; all z
$v = 0$	$y = 6h$; all x, z
$w = 0$	$z = 0$; all x, y
$\frac{\Delta u}{\Delta x} = \bar{\epsilon}_x$	all x, y, z
$\frac{\Delta v}{\Delta x} = 0$	$0 \leq y \leq 6h$; all x, z
$\frac{\Delta v}{\Delta z} = 0$	$x = 0$; $y = \frac{23h}{4}$; all z
$\frac{\Delta w}{\Delta x} = 0$	$0 < z \leq 2h$; all x, y
$\frac{\Delta w}{\Delta y} = 0$	$x = 0$; $\frac{23h}{4} \leq y \leq 6h$; $0 < z \leq 2h$

*u, v, w are displacements in x, y, z coordinates respectively

+h is ply thickness

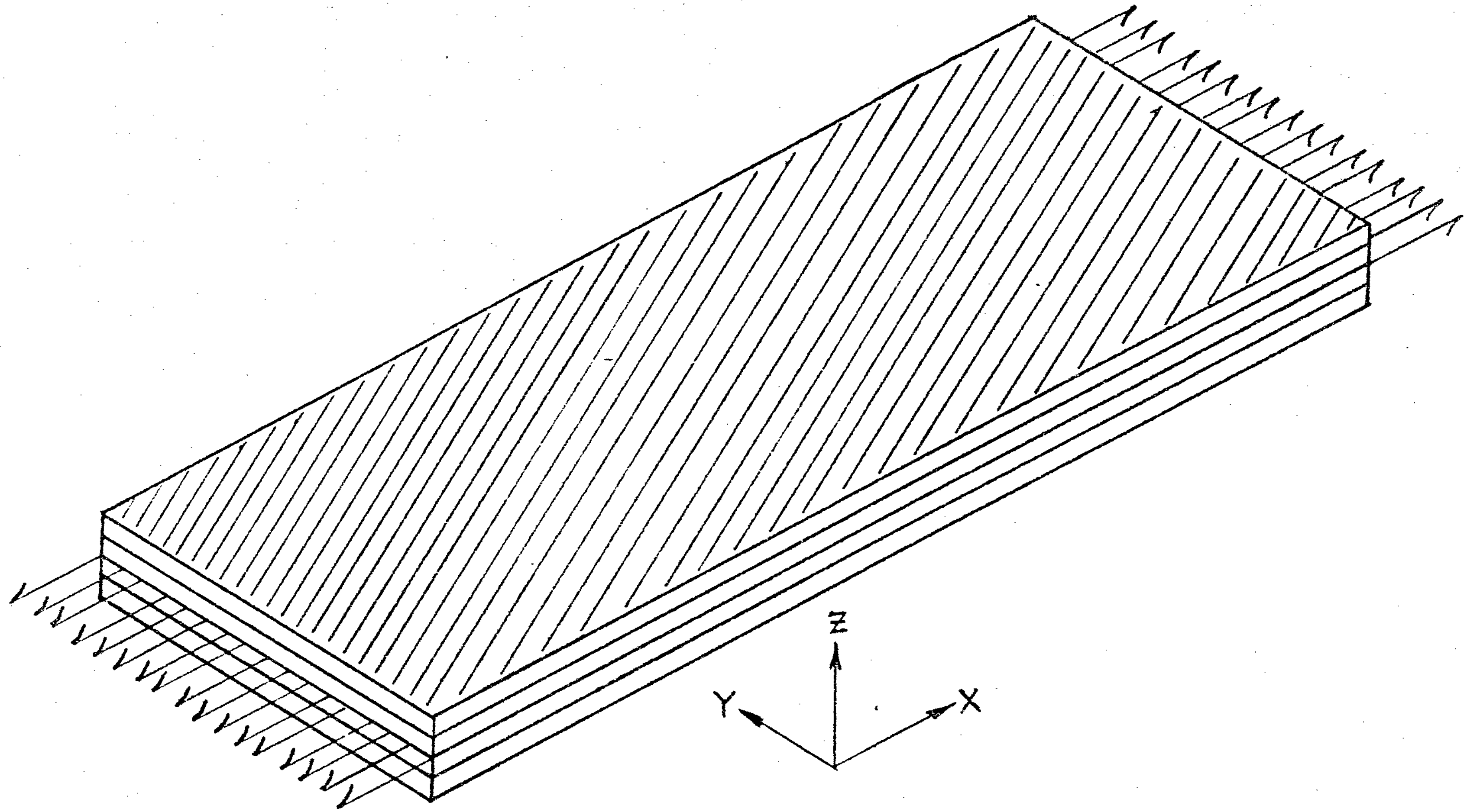

FIGURE 1. FOUR-PLY ±45° LAMINATE IN TENSION

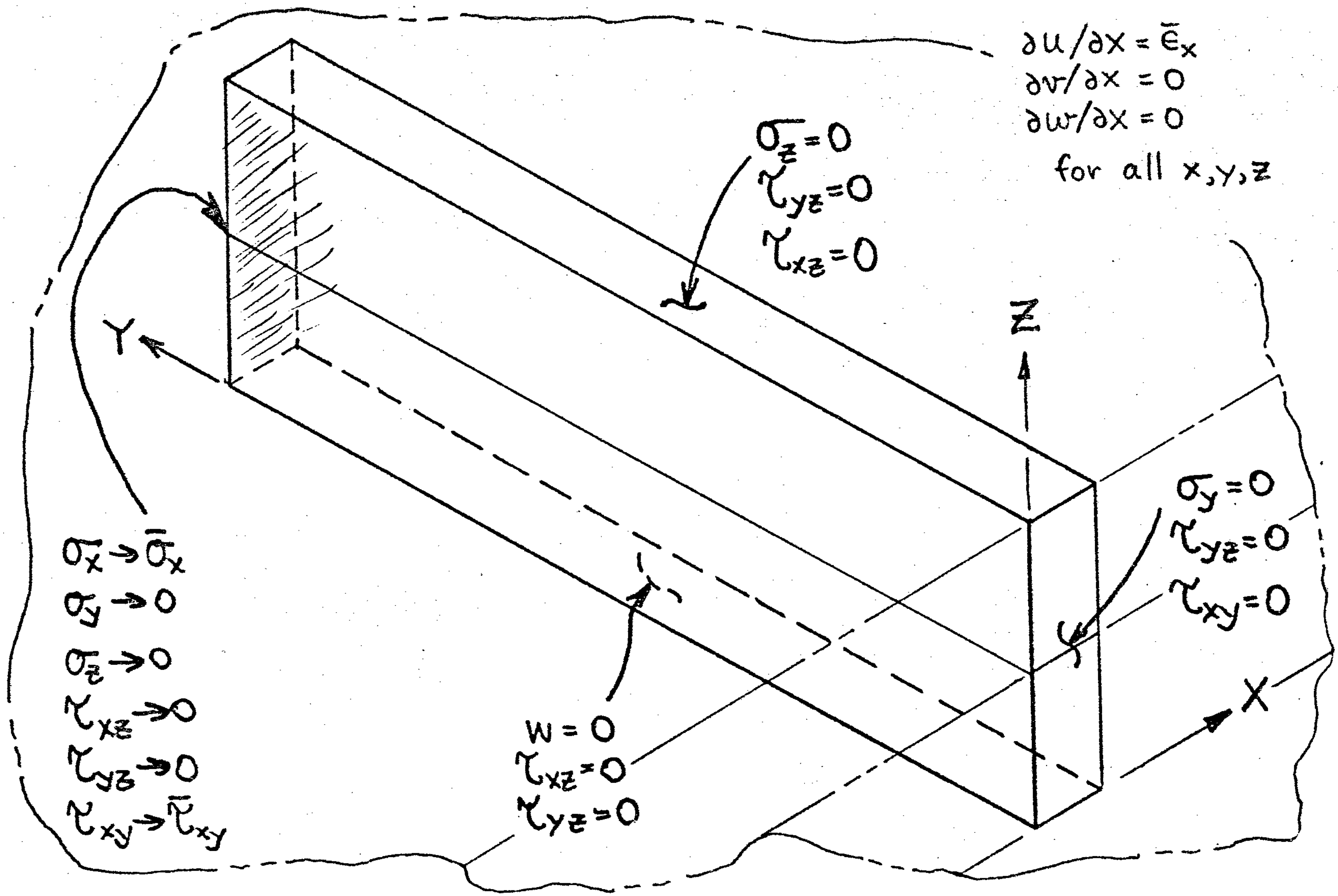

FIGURE 2. REGION TO BE ANALYZED, SHOWING STRESS, STRAIN AND DISPLACEMENT CONDITIONS

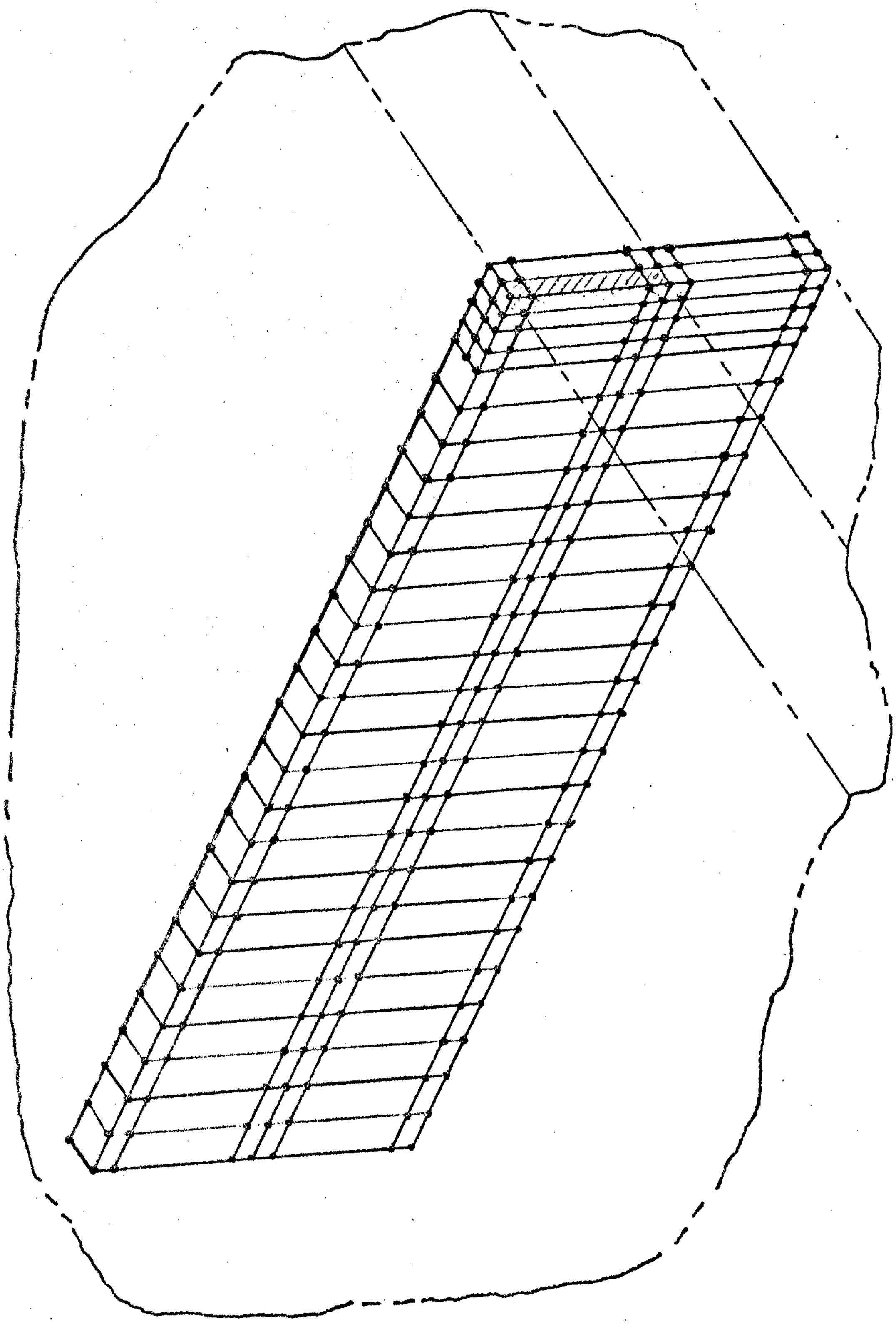

FIGURE 3. FINITE ELEMENT MODEL

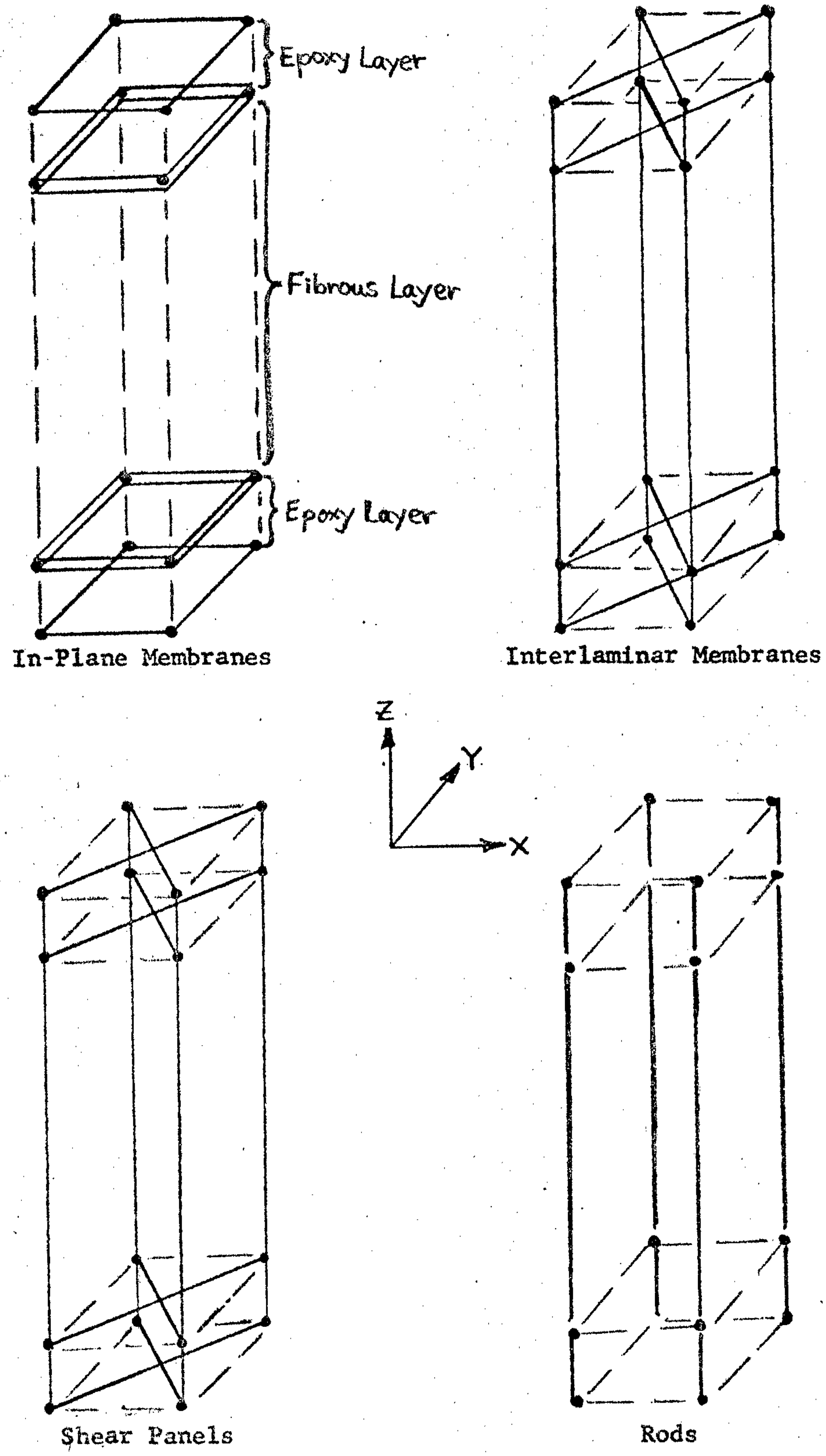

FIGURE 4. ELEMENT ASSEMBLAGE FOR A SINGLE PLY

$$
\begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{vmatrix}
=
\begin{vmatrix}
C_{11} & C_{12} & C_{13} & & & \\
C_{21} & C_{22} & C_{23} & & & \\
C_{31} & C_{32} & C_{33} & & & \\
 & & & C_{44} & & \\
 & & & & C_{55} & \\
 & & & & & C_{66}
\end{vmatrix}
\times
\begin{vmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{vmatrix}
$$

FIGURE 5. ORTHOTROPIC MATERIAL STRESS-STRAIN RELATIONS (IN MATERIAL COORDINATES)

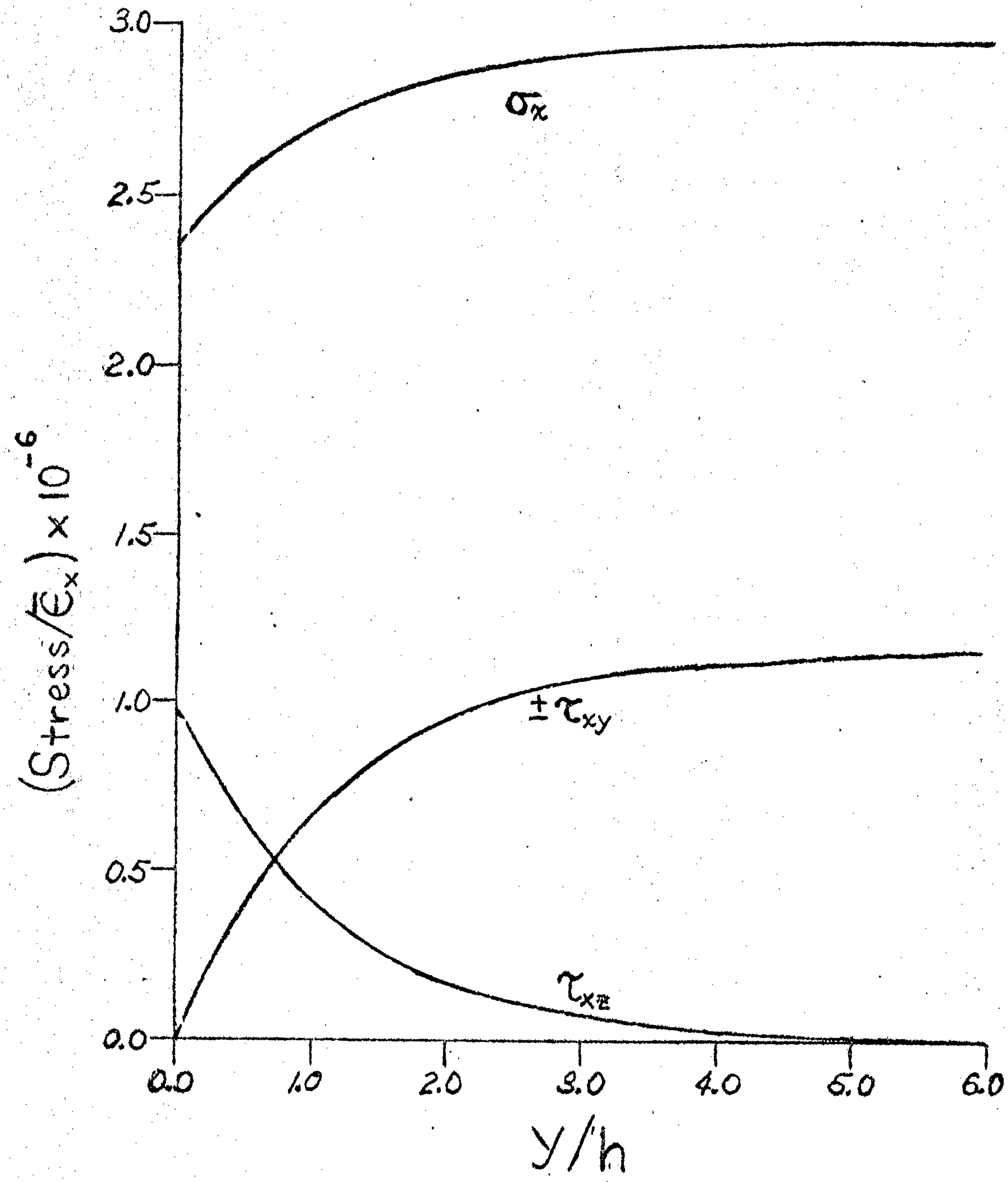

FIGURE 6. FINITE ELEMENT ANALYSIS RESULTS FOR ±45° LAMINATE IN TENSION STRESS (IN LAMINATE COORDINATES) VS. DISTANCE FROM EDGE

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